MINISTRY OF EDUCATION AND TRAINING
NATIONAL ECONOMICS UNIVERSITY

FACULTY OF FUNDAMENTAL SCIENCES

**FINAL EXAMINATION**

**Algebra and calculus:**

**Applications in Economic and Business**

**Mode of study: Full time Intake:**

**Exam place: Exam date:**

**Exam time: Allowed time: *90 minutes***

**Version: 01**

**Question 1:** ***(4.0 points)*** Given three matrices $A, B$ and $C$:

$$A = \begin{bmatrix} -1 & 3 & -2 \\ 2 & -4 & 1 \\ 3 & -1 & -1 \end{bmatrix}; B = \begin{bmatrix} -1 & 1 & -3 & 1 \\ 3 & 4 & 1 & -1 \\ 5 & 2 & 4 & 1 \end{bmatrix}; C = \begin{bmatrix} -1 & 2 & -5 & 0 \\ 3 & 1 & 1 & -3 \\ -2 & 4 & 2 & m \\ -5 & 3 & -1 & 0 \end{bmatrix}$$

***a)*** ***(1.5 points)*** Find the entries in the second row of the product $B^T A^T$.

***b)*** ***(1.0 point)*** For what values of $m$, the matrix $C$ is invertible?

***c)*** ***(1.5 points)*** Use the Cramer's rule to solve the system of 3 linear equations in 3 variables with the augmented matrix $B$.

**Question 2:** ***(1.0 point)*** Find the absolute maximum and the absolute minimum values (if any) of the function $f(x) = (x^2 - 1)^2$ on the interval $-1 \leq x \leq 2$.

**Question 3:** ***(1.0 point)*** The following table gives the values of a function $f$. Use the table to estimate $\int_1^5 f(x)\,dx$ using eight equal subintervals with left endpoints.

| $x$ | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 2.4 | 2.9 | 3.3 | 3.6 | 3.8 | 4.0 | 4.1 | 3.9 | 3.5 |

**Question 4:** ***(1.0 point)*** A manufacturer produces a particular commodity and he estimates that $q$ units of the commodity will be demanded when the price is $p = 1{,}600 - q^2$ dollars per unit, while the same numbers of units will be supplied by producers when the price is $p = 3400 + 2q^2$ dollars per unit. Find the producers' surplus at equilibrium.

**Question 5:** ***(1.5 points)*** A firm produces and sells two goods A and B. The total profit generated from selling $x$ items of A and $y$ items of B is $P(x, y) = 2xy - 3x^2 - 2y^2 + 18x + 4y + 60$. What should $x$ and $y$ be to maximize profit of the firm?

**Question 6:** ***(1.5 points)*** Use method of Lagrange multiplier to find the maximum and minimum values of $f(x, y) = 3y - 4x$ subject to the constraint $x^2 + y^2 = 25$.

**Note:** *No electronic devices, books or notes of any form are allowed.*