



Version: 01

Question 1: (4.0 points) Given a linear system

$$\begin{cases} x - y - 2z = -1 \\ 2x + 3y + z = -2 \\ 5x + 4y + 2z = 4 \end{cases}$$

- a) Write the system in the matrix form $Ax = b$, for $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Find the inverse of A if it exists.
- b) Find the solution to the system.

Question 2: (2.0 points) Find the entries in the second column of the product $B^T A^T$ where

$$A = \begin{bmatrix} -2 & 1 & 3 \\ 4 & -3 & 2 \\ 3 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -2 & 4 \\ -1 & 3 & 2 \\ 4 & 5 & 1 \end{bmatrix}$$

Question 3: (4.0 points) Given a matrix

$$C = \begin{bmatrix} 3 & 2 & -1 & -3 \\ -1 & 1 & 3 & 2 \\ 2 & -4 & 2 & m \\ 4 & 1 & 2 & -1 \end{bmatrix}$$

- a) Find the determinant of the matrix C.
- b) For what values of m , the matrix C is invertible? Then find the entry in the 4th row and the 1st column of C^{-1} .



Version: 02

Question 1: (2.0 points) If $F(x) = f(g(x))$ where $f(-2) = 8$, $f'(-2) = 4$, $g(5) = -2$ and $g'(5) = 6$. Find $F'(5)$.

Question 2: (2.0 points) A psychologist measures a child's capability to learn and remember by the function $f(x) = \frac{\ln(x+1)}{x+1}$ where x is the child's age in years, for $0 \leq x \leq 5$. At what age does a child have the greatest learning capacity?

Question 3: (2.0 points) Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 + 2y^3 = \frac{3}{xy}$.

Question 4: (2.0 points) Find intervals of increase and decrease for the function $f(x) = 3x \sqrt[3]{(5-x)^2}$. Determine whether each critical number corresponds to a relative maximum, a relative minimum, or neither.

Question 5: (2.0 points) Determine where the graph of the functions $f(x) = \frac{3x^2+5}{x^2+3}$ is concave upward and concave downward. Find the x coordinate of each point of inflection.